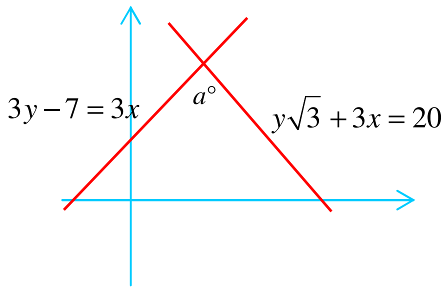
Banker Short Question: No. 1

a) Find the equation of the line through (3, -2) parallel to the line $2y = 3x + 5$

b) Find the equation of the line through (-1, -3) perpendicular to the line $3y - x + 7 = 0$

c) Find the angle that the line $5y + 2x + 3 = 0$ makes with the positive direction of the x-axis

d) Find the angle marked a in the diagram, between the lines $3y - 7 = 3x$ and $y\sqrt{3} + 3x = 20$



[Scroll to next page to see solutions]

Banker Short Question: No. 1

Solution.

a) Arrange the line into normal form to extract the gradient.

$$y = \frac{3}{2}x + \frac{5}{2} \quad \text{hence} \quad \text{gradient} = \frac{3}{2}$$

Line parallel has same gradient.

So line is: $y - (-2) = \frac{3}{2}(x - 3)$

Simplify:

$$y + 2 = \frac{3}{2}(x - 3) \quad \rightarrow \quad 2y + 4 = 3x - 9$$

$$\rightarrow \quad 2y - 3x + 13 = 0$$

b) Arrange the line into normal form to extract the gradient.

$$3y = x - 7 \quad \rightarrow \quad y = \frac{1}{3}x - \frac{7}{3}$$

Hence gradient is $\frac{1}{3}$

Gradient of perpendicular line: -3

(since $m_2 = -\frac{1}{m_1}$)

So line is: $y - (-3) = -3(x - (-1))$

Simplify:

$$y + 3 = -3(x + 1)$$

$$y + 3 = -3x - 3$$

$$y + 3x + 6 = 0$$

c) Arrange to normal form:

$$5y = -2x - 3 \quad \rightarrow \quad y = -\frac{2}{5}x - \frac{3}{5}$$

hence gradient is: $-\frac{2}{5}$

Using $m = \tan\theta$

$$\tan\theta = -\frac{2}{5} \qquad \text{acute } \theta = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$$

since tan is negative then θ must be obtuse.

Hence $\theta = 180 - 21.8 = 158.2°$

[Scroll to next page to see solution for part (d)]

Notes on solution

You must have the equation in normal form:

$$y = mx + c$$

before you can extract the gradient.

Parallel lines have the same gradient.

Use formula:

$$y - b = m(x - a)$$

Any simplified form will do unless form is specified.

Multiply to get rid of fractions.

You must have the equation in normal form:

$$y = mx + c$$

before you can extract the gradient.

Say what you are doing here.

Make reference to $m_2 = -\frac{1}{m_1}$

Use formula:

$$y - b = m(x - a)$$

Any simplified form will do unless form is specified.

Multiply to get rid of fractions.

You must have the equation in normal form:

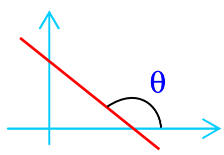
$$y = mx + c$$

before you can extract the gradient.

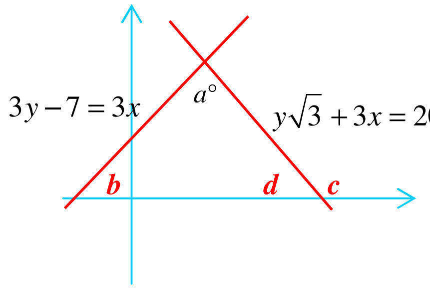
Use: $m = \tan\theta$

Remember if m is negative, then tan θ is negative and so the angle is obtuse.

The angle is **ALWAYS** the anticlockwise angle between the line and the x-axis.



d)



Find gradient of $3y-7=3x$

Rearrange: $3y=3x+7 \;\rightarrow\; y=x+\frac{7}{3}$

Hence gradient = 1

Using $m=\tan\theta \;\rightarrow\; \tan\theta=1$

So angle marked ***b*** on diagram = 45°

You must have the equation in normal form:

$$y=mx+c$$

before you can extract the gradient.

Find gradient of $y\sqrt{3}+3x=20$

Rearrange: $y\sqrt{3}=-3x+20$

$$y=-\frac{3}{\sqrt{3}}x+\frac{20}{\sqrt{3}}$$

So: $y=-\sqrt{3}\,x+\frac{20}{\sqrt{3}}$

Hence gradient is: $-\sqrt{3}$

Surds and gradients often indicate the table of exact values.

When you encounter surds the following note is always useful.

Note that: $-\frac{3}{\sqrt{3}} = -\frac{\sqrt{3}\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$

Using $m=\tan\theta \;\rightarrow\; \tan\theta=-\sqrt{3}$

So acute θ = 60°, but tangent is negative, so θ is obtuse.

Hence θ = 180 – 60 = 120°,

So angle marked ***c*** on diagram = 120°

Furthermore, angle ***d*** on the diagram must be 60°.

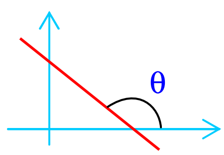
So using the fact that angles in a triangle add up to 180°, then:

$$a = 180-45-60 = 75°$$

Use: $m=\tan\theta$

Remember if m is negative, then tan θ is negative and so the angle is obtuse.

The angle is **ALWAYS** the anticlockwise angle between the line and the x-axis.



Angles in a triangle add up to 180

Use the diagram to clarify your thoughts.