Banker Short Question: No. 2

a) Write x^2-6x+5 in the form $a(x+b)^2+c$

and hence state the least value of this expression

and the value of x for which it occurs.

b) Write $2x^2+8x-3$ in the form $a(x+b)^2+c$

and hence find the maximum value of the fraction

$\frac{3}{2x^2+8x-3}$ and the value of x for which it occurs.

c) Write $9+2x-x^2$ in the form $a(x+b)^2+c$

and hence find the minimum value of the fraction

$\frac{2}{9+2x-x^2}$ and the value of x for which it occurs.

[Scroll to next page to see solution]

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Solution.

a) Write $x^2 - 6x + 5$ in the form $a(x+b)^2 + c$

and hence state the least value of this expression

and the value of x for which it occurs.

$$x^2 - 6x + 5 \rightarrow (x-3)^2 - 9 + 5$$

$$\rightarrow (x-3)^2 - 4$$

Minimum value is -4, when $x = 3$

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b) Write $2x^2 + 8x - 3$ in the form $a(x+b)^2 + c$

and hence find the maximum value of the fraction

$\dfrac{3}{2x^2 + 8x - 3}$ and the value of x for which it occurs.

$$2x^2 + 8x - 3 \rightarrow 2(x^2 + 4x) - 3$$

$$\rightarrow 2\left((x+2)^2 - 4\right) - 3$$

$$\rightarrow 2(x+2)^2 - 8 - 3$$

$$\rightarrow 2(x+2)^2 - 11$$

Minimum value is -11, when $x = -2$

$\dfrac{3}{2x^2 + 8x - 3}$ will have a **maximum** value

when its denominator is a **minimum**.

So fraction can be written as : $\dfrac{3}{2(x+2)^2 - 11}$

So maximum value of fraction is: $-\dfrac{3}{11}$ when $x = -2$

[Scroll to next page to see solution for part (c)]

Notes on solution

Completing the square:

Half the coefficient of x.

Subtract this value squared.

If you expand this you will see it is the same.

The only part that can vary is the $(x-3)^2$.
This is a squared term, so it is always positive.
Its least value is therefore 0.
This occurs when $x = 3$. So least value is -4

Before you can complete the square, the coefficient of x^2 must be +1.

Re-arrange the equation to this form by taking a factor of 2 out of the x^2 and x terms only.

Note that the denominator of the fraction is the expression on which you have just completed the square.

The maximum value of a fraction occurs when its denominator is as small as it can be.

The minimum value of a fraction occurs when its denominator is as large as it can be.

Once you have the minimum value of the denominator, put it into the fraction, to calculate the maximum value of the fraction.

c) Write $9+2x-x^2$ in the form $a(x+b)^2+c$

and hence find the minimum value of the fraction

$\dfrac{2}{9+2x-x^2}$ and the value of x for which it occurs.

Before you can complete the square, the coefficient of x^2 must be +1.

Rearrange expression to normal polynomial form:

$$9+2x-x^2 \rightarrow -x^2+2x+9$$

Re-arrange the equation to a normal polynomial form in decreasing powers of x.

Take out -1 as a factor in the x^2 and the x terms only.

$$\rightarrow -1(x^2-2x)+9$$

Take a factor of -1 out of the x^2 and x terms only.

Now complete the square on the bracket.

$$\rightarrow -1((x-1)^2-1)+9$$

Simplify and rearrange

$$\rightarrow -(x-1)^2+1+9$$

$$\rightarrow 10-(x-1)^2$$

Clearly, the maximum value is 10 when $x = 1$

Note that the denominator of the fraction is the expression on which you have just completed the square.

Looking at the fraction

$\dfrac{2}{9+2x-x^2}$ will have a **minimum** value

when its denominator is a **maximum**.

The minimum value of a fraction occurs when its denominator is as large as it can be.

The maximum value of a fraction occurs when its denominator is as small as it can be.

So fraction can be written as : $\dfrac{2}{10-(x-1)^2}$

So maximum value of fraction is: $\dfrac{2}{10}$ when $x = 1$

Once you have the minimum value of the denominator, put it into the fraction, to calculate the maximum value of the fraction.