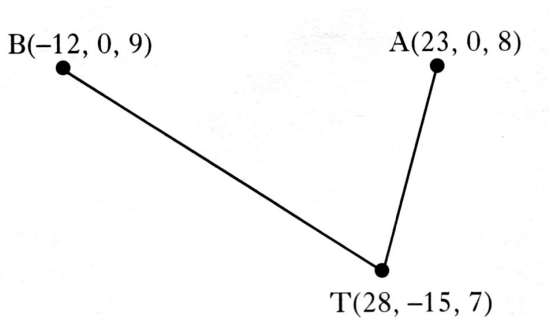
Banker Question: No. 1

The sketch shows the positions of two aircraft A and B and their target T.

Relative to a suitable origin, the coordinates (in miles) of these positions are A(23, 0, 8) B(−12, 0, 9) and T(28, −15, 7).

The aircraft are using GPS to align on their target.



a) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form.

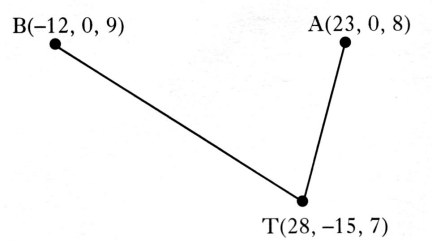
b) Which aircraft is nearer the target ?

c) What is the angle between the two aircract as seen from the target T i.e. what is angle BTA ?

[Scroll to next page to see solution]

Banker Question: No. 1

Solution.



a) $\overrightarrow{TA} = \boldsymbol{a} - \boldsymbol{t} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$

$\overrightarrow{TB} = \boldsymbol{b} - \boldsymbol{t} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$

b) $\left|\overrightarrow{TA}\right| = \sqrt{(-5)^2 + 15^2 + 1^2} = \sqrt{251}$ = 15.8 miles

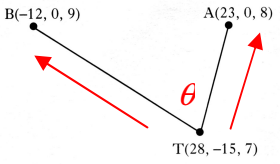
$\left|\overrightarrow{TB}\right| = \sqrt{(-40)^2 + 15^2 + 2^2} = \sqrt{1829}$ = 42.8 miles

So aircraft A is closer to the target.

c) $\cos\theta = \dfrac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$

So in this question – selecting appropriate vectors:

$\cos\theta = \dfrac{\overrightarrow{TA}.\overrightarrow{TB}}{\left|\overrightarrow{TA}\right|\left|\overrightarrow{TB}\right|}$



all we need to work out is the scalar product.

$\overrightarrow{TA}.\overrightarrow{TB} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix} = (-5)\times(-40) + 15\times 15 + 1\times 2 = 427$

$\cos\theta = \dfrac{427}{\sqrt{251}\sqrt{1829}} = 0.6302$

Hence $\theta = \cos^{-1}(0.6302) = 50.9°$

Notes on solution

By looking at the question you should know immediately it is about vectors.

On further reading you should recognise **'the angle between two vectors'**

Remember the position vector of A is just the coordinates of point A.

Finding distance is the same as finding the length of the vector joining the two points.

if $\overrightarrow{PQ} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ then $\left|\overrightarrow{PQ}\right| = \sqrt{x^2 + y^2 + z^2}$

Here is the angle between two vectors. Make sure you know the formula or can work it out by rearranging the one given on the formula sheet.

Make sure your vectors point OUT of the vertex.

Decide what is $\boldsymbol{a}$ and $\boldsymbol{b}$ in this particular question.

$\boldsymbol{a} = \left|\overrightarrow{TA}\right|$ and $\boldsymbol{b} = \left|\overrightarrow{TB}\right|$

Check you know how to find scalar product.

$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ then $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$

Be careful with the division – use brackets.
$427 \div (\sqrt{251} \times \sqrt{1829})$