

Banker Question: No. 10

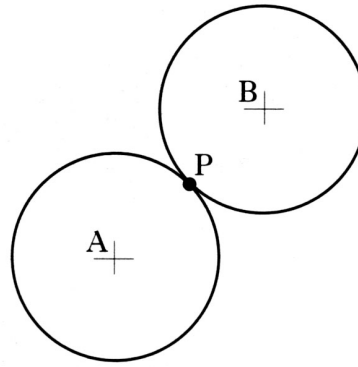
Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$$x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and}$$

$$x^2 + y^2 - 6x - 12y + 20 = 0$$

- a) Find the coordinates of P.
- b) Find the length of AB



(3)

(2)

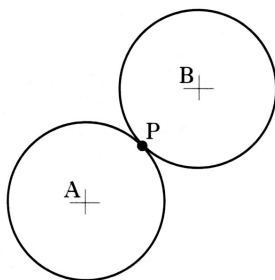
[Scroll to next page to see solution]

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Solution.

$$x^2 + y^2 + 6x + 4y - 12 = 0$$

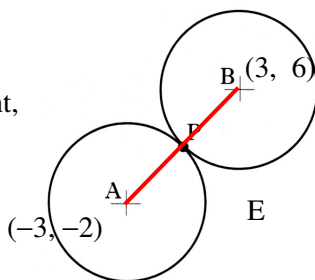
$$x^2 + y^2 - 6x - 12y + 20 = 0$$



a) Centre of first circle: $(-3, -2)$

Centre of 2nd circle: $(3, 6)$

Since the circles are congruent, their radii must be the same, so P lies in the middle of AB.



P is the mid-point of AB.

$$P \text{ is } \left(\frac{-3+3}{2}, \frac{-2+6}{2} \right) \rightarrow P(0, 2)$$

b) We can use the **distance formula** (or Pythagoras) to find AB

$$AB = \sqrt{(3 - (-3))^2 + (6 - (-2))^2}$$

$$AB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

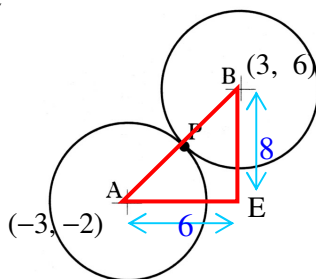
Alternatively,

Draw a right angled triangle and use Pythagoras:

$$AB = \sqrt{6^2 + 8^2}$$

$$AB = \sqrt{36 + 64}$$

$$AB = \sqrt{100} = 10$$



Notes on solution

By looking at the question you should know immediately it is about circles and will involve the centres.

You should recognise the term **congruent** – meaning – same shape – same size i.e. **identical**

A good start in circle questions is to find the centre(s) (and the radius if it is required)

Look for any examples of symmetry.

Always use a diagram and **mark on everything** you know - i.e. coordinates of points, you will then see where to go next.

You could also have worked out the radius of one of the circles using:

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{3^2 + 6^2 - 20}$$

$$r = \sqrt{9 + 36 - 20} = \sqrt{25} = 5$$

and since the circles are congruent, this is half the distance between the centres AB.

So $AB = 10$