Banker Question: No. 11

Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

a) Find expressions for:

i) $f(g(x))$

ii) $g(f(x))$ **(2)**

b) Solve $2f(g(x)) = g(f(x))$ *for* $0 \leq x \leq 360$ **(5)**

[Scroll to next page to see solution]

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Solution.

a) $f(x) = \sin(x^\circ)$ and $g(x) = 2x$

i) $f\left(g(x)\right) = f\left(2x\right) = \sin 2x$

ii) $g\left(f(x)\right) = g\left(\sin x\right) = 2\sin x$

b) Solving $2f(g(x)) = g(f(x))$ *for* $0 \le x \le 360$

$2\sin 2x = 2\sin x$

Rearrange

$2\sin 2x - 2\sin x = 0$

Simplify, by dividing by 2

$\sin 2x - \sin x = 0$

Replace $\sin 2x$ with $2\sin x\cos x$

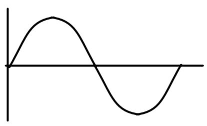
$2\sin x\cos x - \sin x = 0$

Take out $\sin x$ as a factor

$\sin x\left(2\cos x - 1\right) = 0$

Hence: $\sin x = 0$

$x = 0,\ 180,\ 360^\circ$

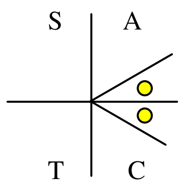


or

$2\cos x - 1 = 0 \quad \Rightarrow \quad \cos x = \frac{1}{2}$

So, acute $x = 60^\circ$

Since cosine is positive
Use ASTC



Hence $x = 60,\ 300^\circ$

All solutions are: $x = 0,\ 60,\ 180,\ 300,\ 360^\circ$

Notes on solution

Replace argument x in $f(x)$ with $g(x)$ i.e. $f(2x)$.

Replace every occurrence of x in the function $f(x)$ with $2x \rightarrow \sin 2x$

Replace argument x in $g(x)$ with $f(x)$ i.e. $g(\sin x)$.

Replace every occurrence of x in the function $g(x)$ with $\sin x \rightarrow 2\sin x$

Rearrange to put the terms equal to 0

Recognise the double angle form sin 2x and look on the inside front cover of your exam paper at the formulae sheet.

This is the common factor form; only two terms and $\sin x$ is common to both.

If $\sin x = 0$, 1 or -1 then sketch a graph to determine the solutions.

This comes from the table of exact values. Learn it !

Use ASTC and the fact that the cosine is positive to get all the solutions.

State all your solutions at the end, clearly!