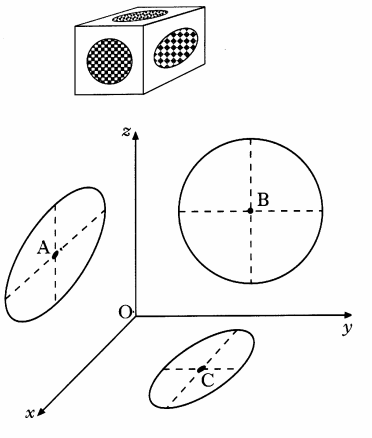
Supplementary Banker Question: No. 1

A box in the shape of a cuboid designed with circles of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid.

The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0)

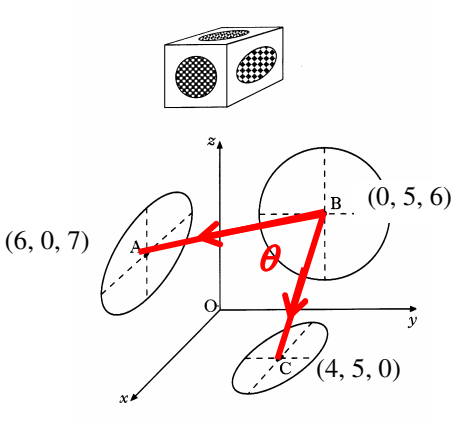
Find the size of angle ABC. **(7)**



[Scroll to next page to see solution]

Supplementary Banker Question: No. 1

Solution.



Find the size of angle ABC.

So: $\cos\theta = \dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}$

Find the two vectors pointing out of the vertex.

$$\overrightarrow{BA} = a - b = \begin{pmatrix}6\\0\\7\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \begin{pmatrix}6\\-5\\1\end{pmatrix}$$

$$\overrightarrow{BC} = c - b = \begin{pmatrix}4\\5\\0\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \begin{pmatrix}4\\0\\-6\end{pmatrix}$$

Scalar Product:

$$\overrightarrow{BA}\cdot\overrightarrow{BC} = \begin{pmatrix}6\\-5\\1\end{pmatrix}\cdot\begin{pmatrix}4\\0\\-6\end{pmatrix} = 24 + 0 - 6 = 18$$

$$\left|\overrightarrow{BA}\right| = \sqrt{6^2 + (-5)^2 + 1^2} = \sqrt{36 + 25 + 1} = \sqrt{62}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{4^2 + 0^2 + (-6)^2} = \sqrt{16 + 0 + 36} = \sqrt{52}$$

Put it all together:

$$\cos\theta = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}$$

$$\rightarrow \quad \cos\theta = \frac{18}{\sqrt{62}\sqrt{52}} = 0.3170 \quad \text{(4dp)}$$

Hence:

$$\theta = \cos^{-1}(0.3170) = 71.51°$$

So angle ABC = 71.5°

Notes on solution

By looking at the question you should know immediately it is about **vectors** and **the angle between two vectors.** The circles are irrelevant.

You should be thinking:

Vectors to point out of the vertex
Components of two vectors
Scalar Product
Length of a vector

formula: $\cos\theta = \dfrac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$

MAKE A SKETCH !

Take care to identify the vectors required and put them into the formula to replace ***a*** and ***b***.

Make sure they point OUT of the vertex.
Make sure you identify the angle correctly.

Find the component form of these two vectors using the position vectors of each end of the lines.

Show your working – do not try and do everything in your head.

Work out the scalar product.

Remember – the result is a **scalar** – **not** a **vector**.

Make sure you know how to do this.

Work out the magnitudes of the vectors.

$\left|\overrightarrow{BA}\right|$ is the length of the line BA.

Square and add the components, he take the square root. Leave in root form

Put it all together taking care with the calculator.

Use brackets around the denominator.

It is better to work out the value of the fraction before using the inverse cosine function.

Enter: $18 \div \left(\sqrt{62} \times \sqrt{52}\right) =$

Round appropriately at the end.

1 decimal place or 3 sig. figs should be adequate.

Make sure you state the answer.