## Banker Question: No. 5

A, B and C have coordinates (-3, 4, 7), (-1, 8, 3) and (0, 10, 1) respectively.

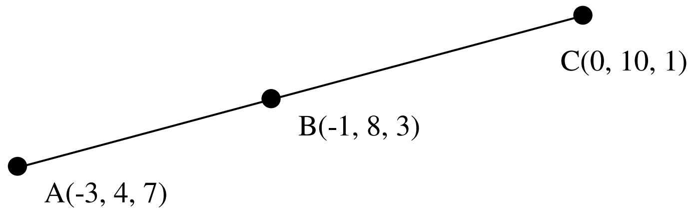
a) Show that A, B and C are collinear. **( 3 )**

b) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$ **( 2 )**

[Scroll to next page to see solution]

## Banker Question: No. 5

## Solution.




a) Consider the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\overrightarrow{AB} = b - a = \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$\overrightarrow{AC} = c - a = \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

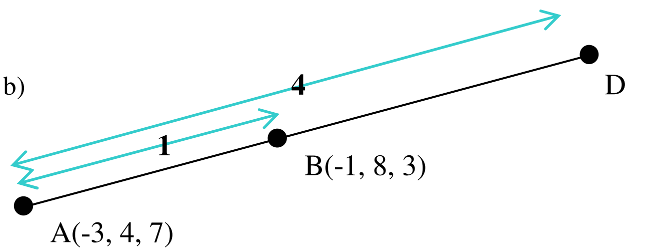
Hence $3\overrightarrow{AB} = 2\overrightarrow{AC}$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are **scalar multiples**

So AB is **parallel** to AC.

Since A is a **common point**, then A, B and C are **collinear**.

b)




$$\overrightarrow{AD} = 4\overrightarrow{AB}$$

$$\boldsymbol{d} - \boldsymbol{a} = 4(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{d} - \boldsymbol{a} = 4\boldsymbol{b} - 4\boldsymbol{a}$$

$$\boldsymbol{d} = 4\boldsymbol{b} - 3\boldsymbol{a}$$

$$\boldsymbol{d} = 4\begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - 3\begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 32 \\ 12 \end{pmatrix} - \begin{pmatrix} -9 \\ 12 \\ 21 \end{pmatrix} = \begin{pmatrix} 5 \\ 20 \\ -9 \end{pmatrix}$$

Hence coordinates are: D(5, 20, -9)

Scroll down for an alternative solution to part b)

### Notes on solution

By looking at the question you should know immediately it is about vectors, collinear points and points dividing lines in ratios.

You should recognise the term:

**collinear**

You should be thinking:
Show vectors are scalar multiples – so parallel – common point – so collinear.

**MAKE A SKETCH**

Choose two vectors that encompass all three points. The vectors should point the SAME way.

You could choose $\overrightarrow{AB}$ and $\overrightarrow{BC}$

or $\overrightarrow{AC}$ and $\overrightarrow{BC}$

or $\overrightarrow{AB}$ and $\overrightarrow{AC}$

The only difference will be which is the common point.

We need to show that these two vectors are scalar multiples. Take out a common factor as the multiplier and you should be left with the same underlying base vector.

If two vectors are scalar multiples of the same vector, then the lines representing these two vectors are parallel.

If in addition, there is a common point, i.e. both vectors contain the same letter (in this case A), then the points must line up, and so they are collinear.

Mark the ratio clearly on the diagram.
In this case we are given the vector form. If we were not, then we would form a ratio using the lengths of the lines (not vectors).

i.e. $\frac{AB}{AD} = \frac{1}{4}$ then cross multiply and change to vector form. $4\overrightarrow{AB} = \overrightarrow{AD}$

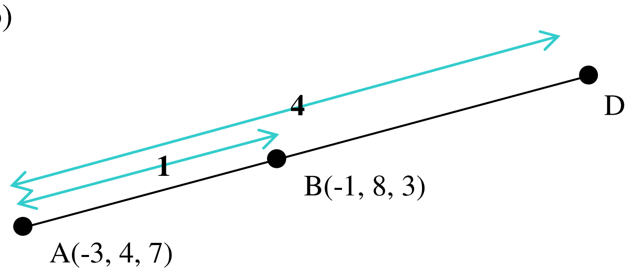
Simplify and rearrange to get the position vector of the point required – in this case, $\boldsymbol{d}$ on its own.

Substitute the column vectors for the position vectors – remember these are simply the coordinates of the point, in column vector form.

Remember to change back to coordinate form as that is what the question asks for. Do not leave as a column vector – you will lose a mark for this.

## Alternative solution to part b)

b)




From previous part:

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$$

Since $\overrightarrow{AD} = 4\overrightarrow{AB}$ then $\overrightarrow{AD} = 4\begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$

This is the displacement vector of D from A.

Hence $\boldsymbol{d} = \boldsymbol{a} + \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix} = \begin{pmatrix} 5 \\ 20 \\ -9 \end{pmatrix}$

So coordinates of D are: D(5, 20, -9)

This time we are looking at a vector as a displacement.

Since the length of AD is 4 × AB and in the same direction, then the displacement from A is: $4\overrightarrow{AB}$

i.e. $4\begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} \rightarrow \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$

Since this is a displacement, it tells us how to get from A to D. So we add this to the position vector of A (which is a column vector of the coordinates).

Don't forget to switch back to coordinate form at the end for the coordinates of D.

Do not leave as a column vector.