

Banker Question: No. 7

a) Express $3 \cos(x^\circ) + 5 \sin(x^\circ)$ in the form $k \cos(x^\circ - a^\circ)$ where $k > 0$ and $0 \leq a \leq 90$ (4)

b) Hence solve the equation $3 \cos(x^\circ) + 5 \sin(x^\circ) = 4$ for $0 \leq x \leq 90$ (3)

[Scroll to next page to see solution]

Banker Question: No. 7

Solution.

a) Express $3\cos(x^\circ) + 5\sin(x^\circ)$ in the form $k\cos(x^\circ - a^\circ)$ where $k > 0$ and $0 \leq a \leq 90$

Expand

$$k\cos(x^\circ - a^\circ) = k\cos x \cos a + k\sin x \sin a$$

Compare coefficients:

$$k\cos a = 3 \quad k\sin a = 5$$

Square and add:

$$k^2 = 3^2 + 5^2 \rightarrow k^2 = 34 \rightarrow k = \sqrt{34}$$

Divide:

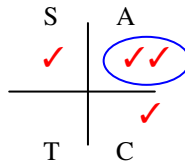
$$\tan a = \frac{5}{3} \quad \text{hence acute } a = \tan^{-1}\left(\frac{5}{3}\right) = 59.03\dots^\circ$$

Determine the quadrant:

sine is + (so quadrants 1 & 2)
cosine is + (so quadrants 1 & 4)

Both are positive in quadrant 1.

$$\text{So, } a = 59^\circ$$



$$\text{Hence: } 3\cos(x^\circ) + 5\sin(x^\circ) = \sqrt{34}\cos(x - 59^\circ)$$

b) Hence solve the equation

$$3\cos(x^\circ) + 5\sin(x^\circ) = 4 \quad \text{for } 0 \leq x \leq 90$$

$$\text{From part a) } 3\cos(x^\circ) + 5\sin(x^\circ) = \sqrt{34}\cos(x - 59^\circ)$$

$$\text{So: } \sqrt{34}\cos(x - 59^\circ) = 4$$

$$\rightarrow \cos(x - 59^\circ) = \frac{4}{\sqrt{34}}$$

$$\text{Hence acute } (x - 59^\circ) = \cos^{-1}\left(\frac{4}{\sqrt{34}}\right) = 46.686\dots^\circ$$

Determine quadrants:

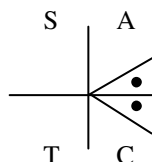
Cosine is +, so (1st & 4th quadrants)

$$x - 59 = 46.7^\circ$$

$$x - 59 = 360 - 46.7^\circ$$

$$\text{So } x = 46.7 + 59 = \mathbf{105.7^\circ}$$

$$x = 360 - 46.7 + 59 = \mathbf{372.3^\circ}$$



Neither of these are in the domain: $0 \leq x \leq 90$

$$\text{Subtract } 360^\circ \text{ and we obtain } 372.3 - 360 = 12.3^\circ$$

The only solution in $0 \leq x \leq 90$ is: $x = 12.3^\circ$

Notes on solution

By looking at the question you should know immediately it is about trigonometry and the wave equation

You should recognise:

$$k\cos(x^\circ - a^\circ)$$

Look on formulae sheet ! Inside front cover.

To compare coefficients you can write:

$$k\cancel{\cos x} \cos a = 3\cancel{\cos x} \rightarrow k\cos a = 3$$

$$k\cancel{\sin x} \sin a = 5\cancel{\sin x} \rightarrow k\sin a = 5$$

Squaring and adding:

$$k^2 \cos^2 a + k^2 \sin^2 a = 3^2 + 5^2$$

$$k^2 (\cos^2 a + \sin^2 a) = 3^2 + 5^2$$

$$\cos^2 a + \sin^2 a = 1 \quad \text{This is always the case}$$

$$\text{Dividing: } \frac{k\sin a}{k\cos a} = \tan a \quad \text{This is always the case.}$$

To determine which quadrant, you have to use:

$$k\cos a = 3 \quad k\sin a = 5$$

You want the quadrant where both of these equations are true. Since sine and cosine are both positive, this only occurs in 1st quadrant.

Re-arrange the equation to solution form:

$$\cos(\dots) = \text{a number.}$$

DO NOT BREAK UP THE ARGUMENT $(x - 59)$ until you have the 2 solutions.

Now you can separate the argument $(x - 59)$.

The domain required is: $0 \leq x \leq 90$

Since neither solution is in this domain, you can add or subtract the period of the wave – in this case 360° to see if you can find a solution within the domain. This is because both the sine and cosine wave are **periodic** functions.