Banker Question: No. 8

Solve the equation $3\cos(2x)+10\cos(x)-1=0$ for $0\le x\le \pi$ correct to 2 decimal places. **(5)**

[Scroll to next page to see solution]

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Solve the equation $3\cos(2x)+10\cos(x)-1=0$
for $0 \le x \le \pi$ correct to 2 decimal places

Solution.

Replace the cos 2x

$3\left(2\cos^2 x-1\right)+10\cos(x)-1=0$

$\rightarrow \quad 6\cos^2 x-3+10\cos(x)-1=0$

$\rightarrow \quad 6\cos^2 x+10\cos(x)-4=0$

Simplify by dividing by 2

$\rightarrow \quad 3\cos^2 x+5\cos(x)-2=0$

This is simply a quadratic equation in cos *x*, so two brackets.

$\rightarrow \quad (3\cos x-1)(\cos x+2)=0$

Hence:

$\rightarrow \quad (3\cos x-1)=0$

or $(\cos x+2)=0$

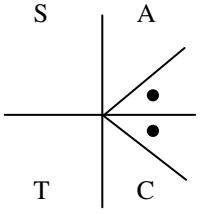
Thus: $\cos x=\frac{1}{3}$ or $\cos x=-2$

discard since $-1 \le \cos x \le 1$

So, if $\cos x=\frac{1}{3}$ then $x=\cos^{-1}\left(\frac{1}{3}\right)$

and so, acute x = 1.23095... **radians**

Now determine quadrants:



Hence:

x = 1.23095... radians

or $x = 2\pi$ - 1.23095.... = 5.0522 ..... radians

Since we only want a solution in the domain $0 \le x \le \pi$

We can discard the second solution.

Solution is: x = 1.23 radians *(2 decimal places)*

Notes on solution

By looking at the question you should know immediately it is about trig equations involving the double angle formula.

You should remember to look inside the front cover of your exam paper for the formulae.

To decide which formula to use for cos 2x, look at the other term in the equation.

If it is a cos x – use $2\cos^2 x$ -1

If it is a sin x – use $1 - 2\sin^2 x$

You will ALWAYS end up with a **quadratic in cos *x*** or **sin *x*.**

If there are 3 terms

– then use **2 brackets**.

If there are two terms

– then use a **common factor**.
(unless it is simply $\cos^2$ x = a number)

Make sure the solutions are possible.

It is often necessary to **DISCARD** one of the values as being out of the range of the sine or cosine.

Check the domain – is it in **degrees** or **radians**.

Set your **calculator** appropriately.

Check the quadrants – using ASTC.

Remember:

180° = π radians and 360° = 2π radians.

Always remember to **check** that your solution is in the **required domain**.

Remember you can always add or subtract the period of the wave (2π in this case) to find any further solutions.

Not necessary here.

Finally, **round your answer** to the accuracy required.