

# Revision Sheet

# Differentiation - DECREASE the power

Rules for Differentiation	
$f(x)$	$f'(x)$
$x^n$	$nx^{n-1}$
$ax^n$	$anx^{n-1}$
$x$	1
$k$	0
$f(x) + g(x)$	$f'(x) + g'(x)$
$(\dots)^n$	$n(\dots)^{n-1} \times \frac{d}{dx}(\dots)$

Rules for Differentiation	
$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

The examples below all use  $f(x)$  and  $f'(x)$  notation, however, the results are the same using  $dy/dx$  notation. Cover up the answers and try them.

function	notes	derivative
$f(x) = 5$	derivative of constant is 0	$f'(x) = 0$
$f(x) = x$	derivative of $x$ is 1	$f'(x) = 1$
$f(x) = 3x$	derivative of $ax$ is $a$	$f'(x) = 3$
$f(x) = x^4$	a basic power of $x$	$f'(x) = 4x^3$
$f(x) = 7x^3$	power of $x$ with multiplier	$f'(x) = 7 \times 3x^2 = 21x^2$
$f(x) = \frac{1}{2}x^2$	power of $x$ with multiplier	$f'(x) = 2 \times \frac{1}{2}x = x$
$f(x) = x^3 - 4x^2 + 5x - 1$	sum of basic powers	$f'(x) = 3x^2 - 8x + 5$
$f(x) = (2x-1)(x+3)$	multiply brackets out $f(x) = 2x^2 + 5x - 3$	$f'(x) = 4x + 5$
$f(x) = (x+1)^2$	multiply brackets out $f(x) = x^2 + 2x + 1$	$f'(x) = 2x + 2$
$f(x) = x^{-3}$	negative power of $x$	$f'(x) = -3x^{-4}$
$f(x) = \frac{1}{x}$	straight line form $f(x) = x^{-1}$	$f'(x) = -1 \cdot x^{-2} \rightarrow -\frac{1}{x^2}$
$f(x) = \frac{1}{x^2}$	straight line form $f(x) = x^{-2}$	$f'(x) = -2x^{-3} \rightarrow -\frac{2}{x^3}$
$f(x) = x^{\frac{1}{2}}$	fractional indices	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$
$f(x) = x^{-\frac{3}{2}}$	negative fractional index	$f'(x) = -\frac{3}{2}x^{-\frac{5}{2}}$
$f(x) = 3x^{-4}$	multiplier with negative index	$f'(x) = -4 \times 3x^{-5} = -12x^{-5}$
$f(x) = 2x^{\frac{2}{3}}$	$f'(x) = \frac{2}{3} \times 2x^{-\frac{1}{3}} \rightarrow \frac{4}{3}x^{-\frac{1}{3}}$	$\rightarrow f'(x) = \frac{4}{3x^{\frac{1}{3}}} \rightarrow \frac{4}{3\sqrt[3]{x}}$

function	notes	derivative
$f(x) = 5x^{-\frac{1}{3}}$	$f'(x) = -\frac{1}{3} \times 5x^{-\frac{4}{3}} \rightarrow \frac{5}{3x^{\frac{4}{3}}}$	$\rightarrow f'(x) = -\frac{5}{3(\sqrt[3]{x})^4}$
$f(x) = \sqrt{x}$	straight line form $f(x) = x^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$
$f(x) = \sqrt[3]{x^2}$	straight line form $f(x) = x^{\frac{2}{3}}$	$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} \rightarrow \frac{2}{3\sqrt[3]{x}}$
$f(x) = \sqrt[4]{x^5}$	straight line form $f(x) = x^{\frac{5}{4}}$	$f'(x) = \frac{5}{4}x^{\frac{1}{4}} \rightarrow \frac{5}{4}\sqrt[4]{x}$
$f(x) = x^{\frac{1}{2}} \left(1 - x^{\frac{1}{2}}\right)$	multiply out $f(x) = x^{\frac{1}{2}} - x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2}} - x$	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 1 \rightarrow \frac{1}{2\sqrt{x}} - 1$
$f(x) = \frac{1}{\sqrt{x}}$	straight line form $f(x) = x^{-\frac{1}{2}}$	$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} \rightarrow -\frac{1}{2(\sqrt{x})^3}$
$f(x) = \frac{3}{2\sqrt{x}}$	str. line form $f(x) = \frac{3}{2} \times \frac{1}{\sqrt{x}} = \frac{3}{2}x^{-\frac{1}{2}}$	$f'(x) = -\frac{1}{2} \times \frac{3}{2}x^{-\frac{3}{2}} \rightarrow -\frac{3}{4(\sqrt{x})^3}$
$f(x) = (1 + \sqrt{x})^2$	multiply out $f(x) = 1 + 2\sqrt{x} + x = 1 + 2x^{\frac{1}{2}} + x$	$f'(x) = \frac{1}{2} \cdot 2x^{-\frac{1}{2}} + 1 \rightarrow \frac{1}{\sqrt{x}} + 1$
$f(x) = \frac{3x^3 - 2x}{x^2}$	split fraction $f(x) = \frac{3x^3}{x^2} - \frac{2x}{x^2} = 3x - 2x^{-1}$	$f'(x) = 3 - (-1)2x^{-2} \rightarrow 3 + \frac{2}{x^2}$
$f(x) = (x+4)^3$	simple chain rule	$f'(x) = 3(x+4)^2 \times 1 \rightarrow 3(x+4)^2$
$f(x) = (2x-3)^4$	chain rule	$f'(x) = 4(2x-3)^3 \cdot 2 \rightarrow 8(2x-3)^3$
$f(x) = 5(3x-1)^3$	chain rule	$f'(x) = 5(3x-1)^2 \cdot 3 \rightarrow 15(3x-1)^2$
$f(x) = (x^3+5)^4$	chain rule	$f'(x) = 2(x^3+5)^3 \cdot 3x^2 \rightarrow 6x^2(x^3+5)^3$
$f(x) = (3x^2-5)^5$	chain rule	$f'(x) = 5(3x^2-5)^4 \cdot 6x \rightarrow 30x(3x^2-5)^4$
$f(x) = 4(2x^3-5)^3$	chain rule	$f'(x) = 3 \times 4(2x^3-5)^2 \cdot 6x^2 \rightarrow 72x^2(2x^3-5)^2$
$f(x) = (3x-1)^{-2}$	chain rule	$f'(x) = -2(3x-1)^{-3} \rightarrow -\frac{2}{(3x-1)^3}$
$f(x) = (x^2+2)^{-1}$	chain rule	$f'(x) = -1(x^2+2)^{-2} \cdot 2x \rightarrow -\frac{2x}{(x^2+2)^2}$
$f(x) = (2x+1)^{\frac{1}{2}}$	chain rule	$f'(x) = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2 \rightarrow \frac{1}{\sqrt{2x+1}}$
$f(x) = (x^2+3)^{\frac{1}{3}}$	chain rule	$f'(x) = \frac{1}{3}(x^2+3)^{-\frac{2}{3}} \cdot 2x \rightarrow \frac{2x}{3(\sqrt[3]{x^2+3})^2}$
$f(x) = (x^2+2x)^3$	chain rule	$f'(x) = 3(x^2+2x)^2 \cdot (2x+2) \rightarrow 3(2x+2)(x^2+2x)^2$
$f(x) = (x^2-x)^{-1}$	chain rule	$f'(x) = -1(x^2-x)^{-2} \cdot (2x-1) \rightarrow -\frac{2x-1}{(x^2-x)^2}$
$f(x) = \frac{1}{(2x-1)}$	S.L.F. C.R. $f(x) = (2x-1)^{-1}$	$f'(x) = -1(2x-1)^{-2} \cdot 2 \rightarrow -\frac{2}{(2x-1)^2}$
$f(x) = \frac{3}{4(2x+5)}$	S.L.F. C.R. $f(x) = \frac{3}{4}(2x+5)^{-1}$	$f'(x) = (-1) \cdot \frac{3}{4}(2x+5)^{-2} \cdot 2 \rightarrow -\frac{3}{2(2x+5)^2}$
$f(x) = \frac{1}{(3x+2)^3}$	S.L.F. C.R. $f(x) = (3x+2)^{-3}$	$f'(x) = -3(3x+2)^{-4} \cdot 3 \rightarrow -\frac{9}{(3x+2)^4}$

function	notes	derivative
$f(x) = \sqrt[3]{(1+6x)}$	$f(x) = (x+1)^{-\frac{1}{2}}$	$f'(x) = -\frac{1}{2}(x+1)^{-\frac{3}{2}} \rightarrow -\frac{1}{2(\sqrt{x+1})^3}$
$f(x) = \frac{3}{\sqrt[3]{9-x}}$	$f(x) = 3(9-x)^{-\frac{1}{3}}$	$f'(x) = -\frac{1}{3} \cdot 3(9-x)^{-\frac{4}{3}} \cdot (-1) \rightarrow \frac{1}{(\sqrt[3]{9-x})^4}$
$f(x) = \left(1 + \frac{1}{x}\right)^4$	$f(x) = (1+x^{-1})^4$	$f'(x) = 4(1+x^{-1})^3 \cdot (-x^{-2}) \rightarrow -\frac{4\left(1+\frac{1}{x}\right)^3}{x^2}$
$f(x) = \sqrt{2x^2 - 4x + 6}$	$f(x) = (2x^2 - 4x + 6)^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}(2x^2 - 4x + 6)^{-\frac{1}{2}} \cdot (4x - 4) \rightarrow \frac{(4x-4)}{2\sqrt{2x^2-4x+6}}$
$f(x) = \frac{1}{\sqrt{x+1}}$	$f(x) = (x+1)^{-\frac{1}{2}}$	$f'(x) = -\frac{1}{2}(x+1)^{-\frac{3}{2}} \cdot 1 \rightarrow -\frac{1}{2(\sqrt{x+1})^3}$
$f(x) = \frac{1}{2x} + \frac{1}{2x-1}$	$f(x) = \frac{1}{2}x^{-1} + (2x-1)^{-1}$	$f'(x) = -\frac{1}{2}x^{-2} + (-1)(2x-1)^{-2} \cdot 2 \rightarrow -\frac{1}{2x^2} - \frac{2}{(2x-1)^2}$
$f(x) = \frac{1}{\sqrt{x+1}}$	$f(x) = (x+1)^{-\frac{1}{2}}$	$f'(x) = -\frac{1}{2}(x+1)^{-\frac{3}{2}} \cdot 1 \rightarrow -\frac{1}{(\sqrt{x+1})^3}$
$f(x) = 3 \sin x$	Basic form with multiplier	$f'(x) = 3 \cos x$
$f(x) = 1 - \sin x$	derivative of 1 is 0	$f'(x) = -\cos x$
$f(x) = 2 + \cos x$	derivative of 2 is 0	$f'(x) = -\sin x$
$f(x) = \frac{1}{2} \cos x$	Basic form with multiplier	$f'(x) = -\frac{1}{2} \sin x$
$f(x) = -2 \cos x$	Basic form with multiplier	$f'(x) = 2 \sin x$
$f(x) = 4 \sin x + 7 \cos x$	f(x) + g(x) with multipliers	$f'(x) = 4 \cos x - 7 \sin x$
$f(x) = 5x^2 - \sin x$	f(x) + g(x) with multipliers	$f'(x) = 10x - \cos x$
$f(x) = \sqrt{x} + \cos x$	f(x) + g(x)	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \sin x \rightarrow \frac{1}{2\sqrt{x}} - \sin x$
$f(x) = \frac{2+x \sin x}{x}$	$f(x) = \frac{2}{x} + \frac{x \sin x}{x} = 2x^{-1} + \sin x$	$f'(x) = -2x^{-2} + \cos x \rightarrow -\frac{2}{x^2} + \cos x$
$f(x) = \frac{3+x^2 \cos x}{x}$	$f(x) = \frac{3}{x^2} + \frac{x^2 \cos x}{x^2} = 3x^{-2} + \cos x$	$f'(x) = -6x^{-3} - \sin x \rightarrow -\frac{6}{x^3} - \sin x$
$f(x) = \sin 2x$	Chain rule	$f'(x) = 2 \cos 2x$
$f(x) = \cos 4x$	Chain rule	$f'(x) = -4 \sin 4x$
$f(x) = \sin \frac{1}{2}x$	Chain rule	$f'(x) = \frac{1}{2} \cos \frac{1}{2}x$
$f(x) = \sin(2x-3)$	Chain rule	$f'(x) = \cos(2x-3) \cdot 2 \rightarrow 2 \cos(2x-3)$
$f(x) = \cos(x^2-1)$	Chain rule	$f'(x) = -\sin(x^2-1) \cdot 2x \rightarrow -2x \sin(x^2-1)$
$f(x) = \frac{1}{\cos x}$	$f(x) = (\cos x)^{-1}$	$f'(x) = -1(\cos x)^{-2}(-\sin x) \rightarrow \frac{\sin x}{\cos^2 x}$

function	notes	derivative
$f(x) = \frac{3}{4 \sin x}$	$f(x) = \frac{3}{4} (\sin x)^{-1}$	$f'(x) = (-1) \frac{3}{4} (\sin x)^{-2} \cos x \rightarrow \frac{-3 \cos x}{4 \sin^2 x}$
$f(x) = \sin^2 x$	$f(x) = (\sin x)^2$	$f'(x) = 2 \sin x \cos x \rightarrow \sin 2x$
$f(x) = \cos^2 x$	$f(x) = (\cos x)^2$	$f'(x) = -2 \cos x \sin x \rightarrow -\sin 2x$
$f(x) = \sin^4 x$	$f(x) = (\sin x)^4$	$f'(x) = 4(\sin x)^3 \cos x \rightarrow 4 \cos x \sin^3 x$
$f(x) = (1 + \sin x)^2$	Chain rule	$f'(x) = 2(1 + \sin x) \cos x \rightarrow 2 \cos x(1 + \sin x)$
$f(x) = \sqrt{1 + \sin x}$	$f(x) = (1 + \sin x)^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}(1 + \sin x)^{-\frac{1}{2}} \cdot \cos x \rightarrow \frac{\cos x}{2\sqrt{1 + \sin x}}$
$f(x) = 2 \sin x \cos x$	$f(x) = \sin 2x$	$f'(x) = 2 \cos 2x$

### Extra Practice Examples from Maths in Action:

<b>Basic differentiation</b>	p. 62 Ex 2: Qu. 1 - 11
<b>Negative and fractional powers</b>	p. 64 Ex 4A: Qu. 1 - 6
<b>Fractions and roots</b>	p. 65 Ex 4B: Qu. 1 - 8
<b>Basic trig functions</b>	p. 221 Ex 2A: Qu. 1 - 4
<b>More trig functions</b>	p. 222 Ex 2B: Qu. 1 - 4
<b>Chain Rule</b>	p. 225 Ex 3A: Qu. 1 - 7
<b>More Chain Rule</b>	p. 226 Ex 3B: Qu. 1 - 5
<b>Chain Rules for Trig</b>	p. 227 Ex 4A: Qu. 1 - 13