Differentiation Techniques

Basic

$y = x^3 - 5x^2 + 4x - 7$

$\frac{dy}{dx} = 3x^2 - 10x + 4$

Differentiation of constant is 0
Differentiation of $kx \to k$
Any multipliers are unaffected

$y = 2x^3 - x - 1$

$\frac{dy}{dx} = 6x^2 - 1$

Differentiation of constant is 0
Differentiation of $-x \to -1$
Any multipliers are unaffected

Breaking brackets

$y = 2x\left(x^3 + 3x - 2\right)$

$y = 2x^4 + 6x^2 - 4x$

$\frac{dy}{dx} = 8x^3 + 12x - 4$

You cannot differentiate in this form.
You must break the brackets.

$y = (2x-1)(x+3)$

$y = 2x^2 + 5x - 3$

$\frac{dy}{dx} = 4x + 5$

You cannot differentiate in this form.
You must break the brackets.

$y = (x+3)^2$

$y = x^2 + 6x + 9$

$\frac{dy}{dx} = 2x + 6$

You cannot differentiate in this form.
You must break the brackets.
(Although you could use the chain rule – Unit 3)

Fractional & Negative Indices

$y = x^{-5}$

$\frac{dy}{dx} = -5x^{-6}$

Usual rule – power to the front, DECREASE power by 1.

$y = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

Usual rule – power to the front, DECREASE power by 1.

Note: $-\frac{1}{2} - 1 \to -\frac{1}{2} - \frac{2}{2} \to -\frac{3}{2}$

$y = x^{-\frac{1}{3}}$

$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}}$

Usual rule – power to the front, DECREASE power by 1.

Note: $-\frac{1}{3} - 1 \to -\frac{1}{3} - \frac{3}{3} \to -\frac{4}{3}$

$y = 8x^{-\frac{3}{4}}$

$\frac{dy}{dx} = -\frac{3}{4} \cdot 8x^{-\frac{7}{4}}$

Usual rule – power to the front, DECREASE power by 1.

Note: $-\frac{3}{4} - 1 \to -\frac{3}{4} - \frac{4}{4} \to -\frac{7}{4}$

Straight Line form

$y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

You cannot differentiate in this form.
You must put into index – straight line form.

$y = \frac{1}{x^3}$

$y = x^{-3}$

$\frac{dy}{dx} = -3x^{-4}$

You cannot differentiate in this form.
You must put into index – straight line form.

$y = \sqrt{x}$

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

You cannot differentiate in this form.
You must put into index – straight line form.

$y = \frac{1}{\sqrt{x}}$

$y = \frac{1}{x^{\frac{1}{2}}} \rightarrow y = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

Remember that $\sqrt{x} = x^{\frac{1}{2}}$

and that $\frac{1}{x} = x^{-1}$

Watch the signs.

Note: $-\frac{1}{2} - 1 \rightarrow -\frac{3}{2}$

Use: $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} \ldots\ldots \; etc.$

$y = \frac{3}{\sqrt{x}}$

$y = 3 \cdot \frac{1}{\sqrt{x}}$

3 is simply a multiplier in this case.
Separate it out, to make conversion to straight line form easier.

$y = 3x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2} \cdot 3x^{-\frac{3}{2}}$

$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{3}{2}}$

Put all the steps in, showing the old index (power) in front).
Then simplify. NO SHORTCUTS.

Differentiation Techniques *continued*

Straight Line form

$$y = \frac{1}{2\sqrt{x}}$$

$$y = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

2 is simply a multiplier in this case, but since it is the denominator it is a multiplier of ½ .

Separate it out, to make conversion to straight line form easier.

$$y = \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \frac{1}{2} x^{-\frac{3}{2}}$$

Put all the steps in, showing the old index (power) in front.

Then simplify. NO SHORTCUTS.

$$\frac{dy}{dx} = -\frac{1}{4} x^{-\frac{3}{2}}$$

$$y = \frac{5}{3\sqrt{x}}$$

$$y = \frac{5}{3} \cdot \frac{1}{\sqrt{x}}$$

The 5 and the 3 are multipliers in this case, but if we separate them out, we find the multiplier is: $\frac{5}{3}$

$$y = \frac{5}{3} \cdot x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \frac{5}{3} x^{-\frac{3}{2}}$$

Put all the steps in, showing the old index (power) in front.

Then simplify. NO SHORTCUTS.

$$\frac{dy}{dx} = -\frac{5}{6} x^{-\frac{3}{2}}$$

$$y = \frac{2}{\sqrt[3]{x^2}}$$

$$y = 2x^{-\frac{2}{3}}$$

The cube root of x^2 is $x^{\frac{2}{3}}$.

$$\frac{dy}{dx} = -\frac{2}{3} \cdot 2x^{-\frac{5}{2}}$$

Note: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$$y = x^{\frac{1}{2}}\left(1 - x^{\frac{1}{2}}\right)$$

$$y = x^{\frac{1}{2}} - x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$$

Multiply out the brackets

$$y = x^{\frac{1}{2}} - x$$

Note: $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^1 = x$
(adding the indices)

$$y = \frac{2x^3 - 5x}{x^2}$$

$$y = \frac{2x^3}{x^2} - \frac{5x}{x^2}$$

Split the fraction.

$$y = 2x - \frac{5}{x}$$

Simplify

$$y = 2x - 5x^{-1}$$

Straight Line form

$$\frac{dy}{dx} = 2 + 5x^{-2}$$

Differentiate

Differentiation Techniques *continued*

Chain Rule

$y=(x+4)^3$

$\frac{dy}{dx}=3(x+4)^2\cdot 1$

Take the power to the front;
Decrease the power by 1.
Multiply by the differential of the bracket.

$y=(2x-3)^5$

$\frac{dy}{dx}=5(2x-3)^4\cdot 2 \quad \rightarrow \quad \frac{dy}{dx}=10(2x-3)^4$

$y=4(3x^2-1)^3$

$\frac{dy}{dx}=3\cdot 4(3x^2-1)^2\cdot 6x^2 \quad \rightarrow \quad \frac{dy}{dx}=72x^2(3x^2-1)^2$

$y=(3x-4)^{-3}$

$\frac{dy}{dx}=-3\cdot(3x-4)^{-4}\cdot 3 \quad \rightarrow \quad \frac{dy}{dx}=-9(3x-4)^{-4}$

$y=(2x+1)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}\cdot(2x+1)^{-\frac{1}{2}}\cdot 2 \quad \rightarrow \quad \frac{dy}{dx}=(2x+1)^{-\frac{1}{2}}$

$y = \frac{1}{2x+1}$

$y = (2x+1)^{-1}$

$\frac{dy}{dx} = -1\cdot(2x+1)^{-2}\cdot 2 \quad \rightarrow \quad \frac{dy}{dx} = -2(2x+1)^{-2}$

$y = \frac{3}{4(2x+5)}$

$y = \frac{3}{4}(2x+5)^{-1}$

$\frac{dy}{dx} = -1\cdot\frac{3}{4}(2x+5)^{-2}\cdot 2 \quad \rightarrow \quad \frac{dy}{dx} = -\frac{6}{4}(2x+5)^{-2}$

Trig Functions

$y=\sin x$ $\qquad \frac{dy}{dx}=\cos x$

$y=-\sin x$ $\qquad \frac{dy}{dx}=-\cos x$

$y=\cos x$ $\qquad \frac{dy}{dx}=-\sin x$

$y=-\cos x$ $\qquad \frac{dy}{dx}=\sin x$

Differentiation Techniques *continued*

Trig Functions

$y = 3\sin x$	$\frac{dy}{dx} = 3\cos x$
$y = x + \cos x$	$\frac{dy}{dx} = 1 - \sin x$
$y = 3x^2 - 2\sin x$	$\frac{dy}{dx} = 6x - 2\cos x$

Chain Rule for Trig Functions

$y = \sin 2x$	$\frac{dy}{dx} = 2\cos 2x$
$y = 2\cos 3x$	$\frac{dy}{dx} = -6\sin 3x$
$y = 3\cos^2 x$	$y = 3(\cos x)^2$ $\frac{dy}{dx} = 2 \cdot 3(\cos x)^1(-\sin x) \quad \rightarrow \quad \frac{dy}{dx} = -6\cos x \sin x$
$y = \frac{5 + x^2 \sin x}{x^2}$	$y = \frac{5}{x^2} + \frac{x^2 \sin x}{x^2}$ $y = 5x^{-2} + \sin x$ $\frac{dy}{dx} = -10x^{-3} + \cos x$

All the examples you will encounter in the examination, can be done by simply combining the few rules you know together with a few techniques to put them into the required form.

- Differentiaton of x^n
- Chain rule for algebraic functions: $$\frac{d}{dx} f(g(x)) = \frac{df}{dg} \cdot \frac{dg}{dx}$$
- Chain rule for trigonometric functions.