

Indices

Rules of Indices:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Examples of using the rules:

1. Multiplying – add the indices

e.g. $x^4 \times x^5 \rightarrow x^9$

2. Dividing – subtract the indices

e.g. $z^7 \div z^5 \rightarrow z^2$ and $\frac{p^6}{p^2} \rightarrow p^4$

3. Power of a power – multiply the indices

e.g. $(x^3)^2 \rightarrow x^6$

Some useful results – Special indices

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|----|---------------------|--|--------------------------|---------------------|
| 1. | $a^0 \rightarrow 1$ | anything raised to the power 0 is 1 | e.g. $3^0 \rightarrow 1$ | $x^0 \rightarrow 1$ |
| 2. | $a^1 \rightarrow a$ | anything raised to the power 1 is itself | e.g. $5^1 \rightarrow 5$ | $x^1 \rightarrow x$ |
| | | note the reverse: | i.e. $m \rightarrow m^1$ | |

Negative and fractional indices

1. $a^{-1} \rightarrow \frac{1}{a}$ and $a^{-2} \rightarrow \frac{1}{a^2}$ in general $a^{-n} \rightarrow \frac{1}{a^n}$ 'minus' - means '1 over'

2. $a^{\frac{1}{2}} \rightarrow \sqrt{a}$ and $a^{\frac{1}{3}} \rightarrow \sqrt[3]{a}$ in general $a^{\frac{1}{n}} \rightarrow \sqrt[n]{a}$

the denominator in the fraction denotes which root it is.

e.g. $\sqrt{16} = 4$ (since $4 \times 4 = 16$) and $\sqrt[3]{27} = 3$ (since $3 \times 3 \times 3 = 27$)

$\sqrt[3]{8} = 2$ (since $2 \times 2 \times 2 = 8$) and $\sqrt[4]{16} = 2$ (since $2 \times 2 \times 2 \times 2 = 16$)

3. $a^{\frac{2}{3}} \rightarrow \sqrt[3]{a^2}$ or $(\sqrt[3]{a})^2$ in general $a^{\frac{m}{n}} \rightarrow \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$

the denominator gives the root, the numerator raises it to the power.

Some examples of applications

1. $(pq)^3$ this means $pq \times pq \times pq \rightarrow p \times q \times p \times q \times p \times q \rightarrow p^3 q^3$

2. $(2a)^4$ this means $2a \times 2a \times 2a \times 2a \rightarrow 16a^4$

3. $2a^4$ this means $2 \times a \times a \times a \times a \rightarrow 2a^4$ do not confuse this with the one above.

4. $\frac{5}{(5p)^3}$ simplify the denominator, then cancel. $\frac{5}{5 \times 5 \times 5 \times p^3} \rightarrow \frac{\cancel{5}^1}{\cancel{5}^1 \times 5 \times 5 \times p^3} \rightarrow \frac{1}{25p^3}$

5. $(ab^3)^2$ this means $ab^3 \times ab^3 \rightarrow a \times b^3 \times a \times b^3 \rightarrow a^2 b^6$

6. $3x^2 \times 5x^2$ deal with numbers first. $\rightarrow 3 \times 5 \times x^2 \times x^2 \rightarrow 15x^4$

7. $12y^3 \div 6y$ deal with numbers first. $12 \div 6 \rightarrow 2$ and $y^3 \div y \rightarrow y^2$ so $\rightarrow 2y^2$

or in fraction form and cancel $\rightarrow \frac{12y^3}{6y} \rightarrow \frac{\cancel{12}^2 \cancel{y}^3 y^2}{\cancel{6}^1 \cancel{y}^1} \rightarrow 2y^2$

8. $w^3(w^4 + w^3)$ Multiply out the brackets in the usual way. $\rightarrow w^3 \times w^4 + w^3 \times w^3 \rightarrow w^7 + w^6$

9. $\frac{u^4 \times u^5}{u^3}$ Numerator first $\rightarrow \frac{u^9}{u^3} \rightarrow u^6$

10. $\frac{s^2 \times s^{-4}}{s^{-3}}$ Numerator first $\rightarrow \frac{s^{-2}}{s^{-3}} \rightarrow s^{-2-(-3)} \rightarrow s^1 \rightarrow s$

11. $5^n = 25$ What power do we raise 5 to get 25, n must be 2, so $n = 2$
 At the moment, take a good guess and check it out. They are usually easy numbers.
 Alternatively, put the right hand side in index form and compare indices.
 $5^n = 5^2$ so clearly $n = 2$

12. $4^n = \frac{1}{4}$ Put the right hand side in index form. $4^n = 4^{-1}$ so clearly $n = -1$

13. $4x^{-5}$ Writing with positive indices, get rid of the '-1' by using '1 over'.
 $\rightarrow 4 \times \frac{1}{x^5} \rightarrow \frac{4}{x^5}$ Note the '-5' power only applies to the x
 Treat the number as a separate multiplier.

14. $\frac{2}{3}v^{-4}$ Writing with positive indices, get rid of the '-1' by using '1 over'.
 Treat the number as a separate multiplier.
 $\rightarrow \frac{2}{3} \times v^{-4} \rightarrow \frac{2}{3} \times \frac{1}{v^4} \rightarrow \frac{2}{3v^4}$

15. $t^{\frac{3}{4}}$ Writing in root form $\rightarrow \sqrt[4]{t^3}$ or $(\sqrt[4]{t})^3$

16. $m^{-\frac{4}{5}}$ Writing in root form $\rightarrow \frac{1}{m^{\frac{4}{5}}}$ (negative index first) $\rightarrow \frac{1}{\sqrt[5]{m^4}}$ or $\frac{1}{(\sqrt[5]{m})^4}$

17. $\sqrt[3]{p^4}$ Writing in index form $\rightarrow p^{\frac{4}{3}}$ (check it goes back to the root form)
Remember: denominator = root, numerator = power.
18. $16^{\frac{1}{4}}$ Evaluate. Put into root form. $\rightarrow \sqrt[4]{16} \rightarrow 2$ then check it. ($2 \times 2 \times 2 \times 2 = 16$)
19. $27^{\frac{2}{3}}$ Evaluate. Put into root form. $\rightarrow (\sqrt[3]{27})^2 \rightarrow 3^2 \rightarrow 9$
 $(\sqrt[3]{27})^2$ is easier to work with than $\sqrt[3]{27^2}$.
20. $v^{\frac{5}{2}} \times v^{\frac{1}{2}}$ Simplify – use the rules. Add the indices. $v^{\frac{5}{2} + \frac{1}{2}} \rightarrow v^{\frac{6}{2}} \rightarrow v^3$
21. $x^{\frac{3}{5}} \div x^{-\frac{1}{5}}$ Simplify – use the rules. Subtract the indices. $x^{\frac{3}{5} - (-\frac{1}{5})} \rightarrow x^{\frac{3}{5} + \frac{1}{5}} \rightarrow x^{\frac{4}{5}}$
22. $\left(p^{\frac{1}{2}}\right)^3$ Simplify – use the rules. Multiply the indices. $\rightarrow p^{\frac{3}{2}}$
23. $(a^2 + 1)(a^{-2} + 1)$ Use FOIL and the rules in the normal way.
 $\rightarrow a^2 \times a^{-2} + a^2 \times 1 + 1 \times a^{-2} + 1 \times 1 \rightarrow a^0 + a^2 + a^{-2} + 1$
 $\rightarrow 1 + a^2 + a^{-2} + 1 \rightarrow 2 + a^2 + a^{-2}$
24. $\left(x^{\frac{1}{2}} + 1\right)\left(x^{\frac{1}{2}} + 1\right)$ Use FOIL and the rules in the normal way.
 $\rightarrow x^{\frac{1}{2}} \times x^{\frac{1}{2}} + x^{\frac{1}{2}} \times 1 + 1 \times x^{\frac{1}{2}} + 1 \times 1 \rightarrow x^{\frac{1}{2} + \frac{1}{2}} + x^{\frac{1}{2}} + x^{\frac{1}{2}} + 1$
 $\rightarrow x^1 + 2x^{\frac{1}{2}} + 1 \rightarrow x + 2x^{\frac{1}{2}} + 1$