

Solutions to Revision Sheet on Indices

1. a. $x^4 \times x^3 = x^7$
 b. $y^5 \times y^2 = y^7$
 c. $p^3 \times p = p^4$
 d. $x^7 \times x^{-3} = x^4$
 e. $x^5 \div x^2 = x^3$
 f. $\frac{y^7}{y^3} = y^4$
 g. $n^{10} \div n^4 = n^6$
 h. $z^6 \div z^3 = z^3$
 i. $x^5 \div x^{-2} = x^7$
 j. $\frac{w^5}{w^{-3}} = w^8$
 k. $p^5 \div p^{-2} = p^7$
 l. $x^3 \div x^{-3} = x^6$
 m. $(x^3)^2 = x^6$
 n. $(y^3)^{-2} = y^{-6}$
 o. $(z^{-1})^{-3} = z^3$
 p. $\left(\frac{1}{w^2}\right)^{-3} = (w^{-2})^{-3} = w^6$
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2. a. $(ab)^4 = a^4b^4$
 b. $(xy)^3 = x^3y^3$
 c. $\frac{3}{(3a)^2} = \frac{3}{9a^2} = \frac{1}{3a^2}$
 d. $(2a)^3 = 8a^3$
 e. $(3y)^4 = 81y^4$
 f. $(mn^2)^5 = m^5n^{10}$
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3. a. $2a^2 \times a^3 = 2a^5$
 b. $x^2 \times 3x^4 = 3x^6$
 c. $2y^2 \times 3y^2 = 6y^4$
 d. $8x^2 \div 2x = 4x$
 e. $10y^3 \div 2y^2 = 5y$
 f. $6z^4 \div 3z^2 = 2z^2$
 g. $10y^2 \div 5y^3 = 2y^{-1} = 2 \cdot \frac{1}{y} = \frac{2}{y}$
 h. $12x^4 \div 3x^{-2} = 4x^6$

4. a. $x^3(x^2 + x^4) = x^5 + x^7$
 b. $y^2(y^3 - y) = y^5 - y$
 c. $z^2(z^4 - 1) = z^6 - z^2$
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5. a. $\frac{x^3 \times x^4}{x^2} = \frac{x^7}{x^2} = x^5$
 b. $\frac{t^3 \times t^6}{t^4} = \frac{t^9}{t^4} = t^5$
 c. $\frac{w^2 \times w^{-3}}{w^{-4}} = \frac{w^{-1}}{w^{-4}} = w^{(-1-(-4))} = w^3$
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6. a. $2^n = 8 \quad n = 3$
 b. $3^n = 81 \quad n = 4$
 c. $4^n = 4 \quad n = 1$
 d. $3^n = \frac{1}{3} \quad 3^n = 3^{-1} \therefore n = -1$
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7. a. $3x^{-1} = 3 \cdot \frac{1}{x} \rightarrow \frac{3}{x}$
 b. $5y^{-3} = 5 \cdot \frac{1}{y^3} \rightarrow \frac{5}{y^3}$
 c. $\frac{1}{2}x^{-1} = \frac{1}{2} \cdot \frac{1}{x} \rightarrow \frac{1}{2x}$
 d. $\frac{3}{4}u^{-3} = \frac{3}{4} \cdot \frac{1}{u^3} \rightarrow \frac{3}{4u^3}$
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8. a. $m^{\frac{3}{4}} = (\sqrt[4]{m})^3 \text{ or } \sqrt[4]{m^3}$
 b. $x^{\frac{4}{5}} = (\sqrt[5]{x})^4 \text{ or } \sqrt[5]{x^4}$
 c. $k^{\frac{2}{3}} = (\sqrt[3]{k})^2 \text{ or } \sqrt[3]{k^2}$
 d. $x^{\frac{1}{2}} = \sqrt{x}$
 e. $x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$
 f. $x^{\frac{2}{3}} = \frac{1}{x^{\frac{1}{3}}} = \frac{1}{(\sqrt[3]{x})^2} \text{ or } \frac{1}{\sqrt[3]{x^2}}$

Solutions to Indices (continued)

9. a. $\sqrt[3]{y^5} = y^{\frac{5}{3}}$

b. $\sqrt[3]{z^4} = z^{\frac{4}{3}}$

c. $\sqrt{x} = x^{\frac{1}{2}}$

d. $\sqrt[3]{t} = t^{\frac{1}{3}}$

10. a. $9^{\frac{1}{2}} = \sqrt{9} = 3$

b. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

c. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

d. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

e. $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

f. $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

g. $25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$

h. $81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = 27$

11. a. $u^{\frac{3}{2}} \times u^{\frac{1}{2}} = u^{\frac{3}{2} + \frac{1}{2}} = u^2 = u^2$

b. $v^{\frac{4}{3}} \times v^{\frac{1}{3}} = v^{\frac{4}{3} + \frac{1}{3}} = v^{\frac{5}{3}} = v^{\frac{5}{3}}$

c. $x^{\frac{3}{2}} \div x^{\frac{1}{2}} = x^{\frac{3}{2} - \frac{1}{2}} = x^1 = x$

d. $y^{\frac{3}{4}} \div y^{\frac{1}{4}} = y^{\frac{3}{4} - \frac{1}{4}} = y^{\frac{2}{4}} = y^{\frac{1}{2}} = \sqrt{y}$

e. $\left(t^{\frac{1}{2}}\right)^2 = t^{2 \times \frac{1}{2}} = t^1 = t$

f. $\left(p^{-\frac{1}{2}}\right)^2 = p^{-\frac{1}{2} \times 2} = p^{-1} = \frac{1}{p}$

g. $\left(q^{\frac{1}{4}}\right)^0 = q^{\frac{1}{4} \times 0} = q^0 = 1$

h. $\left(r^3\right)^{\frac{1}{3}} = r^{3 \times \frac{1}{3}} = r^1 = r$

12a. $x^{\frac{1}{2}} \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$
 $= x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} = x^1 + x^0 = x + 1$

12b. $m^{-\frac{3}{4}} \left(m^{\frac{7}{4}} - m^{-\frac{1}{4}}\right)$
 $= m^{-\frac{3}{4}} \cdot m^{\frac{7}{4}} - m^{-\frac{3}{4}} \cdot m^{-\frac{1}{4}} = m^{\frac{4}{4}} - m^{\frac{4}{4}}$
 $= m^1 - m^{-1} = m - \frac{1}{m}$

12c. $\frac{a^3 \times a^{-2}}{a} = \frac{a^1}{a} = \frac{a}{a} = 1$

12d. $\frac{c^2 \times c^{-2}}{c^{-1}} = \frac{c^0}{c^{-1}} = c^{0 - (-1)} = c^1 = c$

12e. $\frac{x^{\frac{1}{2}} \times x^{-\frac{3}{2}}}{x^2} = \frac{x^{-1}}{x^2} = x^{-1-2} = x^{-3} = \frac{1}{x^3}$

12f. $\frac{y^{-\frac{1}{3}} \times y^{\frac{4}{3}}}{y} = \frac{y^1}{y} = \frac{y^1}{y} = \frac{y}{y} = 1$

12g. $(a^2 + 1)(a^{-2} + 1)$ (Use FOIL)
 $= a^2 \cdot a^{-2} + a^2 + a^{-2} + 1 = a^0 + a^2 + \frac{1}{a^2} + 1$
 $= 1 + a^2 + \frac{1}{a^2} + 1 = 2 + a^2 + \frac{1}{a^2}$

12h. $(b^{-1} + 1)(b^{-1} - 1)$ (Use FOIL)
 $= b^{-1} \cdot b^{-1} - b^{-1} + b^{-1} - 1 = b^{-2} - 1 = \frac{1}{b^2} - 1$

12i. $\left(x^{\frac{1}{2}} + 1\right)\left(x^{\frac{1}{2}} + 1\right)$ (Use FOIL)
 $= x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} + x^{\frac{1}{2}} + 1 = x + 2\sqrt{x} + 1$

12j. $\left(u^{\frac{1}{2}} + 1\right)\left(u^{-\frac{1}{2}} + 1\right)$ (Use FOIL)
 $= u^{\frac{1}{2}} \cdot u^{-\frac{1}{2}} + u^{\frac{1}{2}} + u^{-\frac{1}{2}} + 1 = u^0 + \sqrt{u} + 1 = 2 + \sqrt{u}$