

Surds

What is a surd?

A surd is an irrational number – a number that cannot be expressed as a fraction.

In simple terms – it is the square root of a number that cannot be evaluated as a whole number.

e.g. These are surds: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ etc

These are NOT surds: $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, etc since these are 2, 3, 4, 5 respectively.

Rules of Surds:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

These can be used in both directions

Examples of using the rules:

1. Multiplication rule

e.g. $\sqrt{3} \times \sqrt{2} \rightarrow \sqrt{3 \times 2} \rightarrow \sqrt{6}$

in reverse we can also say: $\sqrt{20} \rightarrow \sqrt{4 \times 5} \rightarrow \sqrt{4} \times \sqrt{5} \rightarrow 2\sqrt{5}$

This is a useful way of simplifying surds.

2. Division rule

e.g. $\frac{\sqrt{24}}{\sqrt{3}} \rightarrow \sqrt{\frac{24}{3}} \rightarrow \sqrt{8}$ Note that we can further simplify this:

$$\sqrt{8} \rightarrow \sqrt{4 \times 2} \rightarrow \sqrt{4} \times \sqrt{2} \rightarrow 2\sqrt{2}$$

Also the other way: $\sqrt{\frac{72}{9}} \rightarrow \frac{\sqrt{72}}{\sqrt{9}} \rightarrow \frac{\sqrt{72}}{3}$

We can also further simplify this: $\frac{\sqrt{72}}{3} \rightarrow \frac{\sqrt{36 \times 2}}{3} \rightarrow \frac{\sqrt{36} \times \sqrt{2}}{3} \rightarrow \frac{6 \times \sqrt{2}}{3} \rightarrow 2\sqrt{2}$

Useful tip: $\sqrt{2} \times \sqrt{2} = 2$ and in general: $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = \sqrt{a^2} = a$

Applications

Simplification:

Simplify: $\sqrt{54}$

To break this up, we look for the largest square number that is a factor.

e.g. $54 = 2 \times 27$ or 9×6 or 3×18

The quickest way is to see which of the **square numbers**

i.e. 4, 9, 16, 25, 36, 49, 64, etc. will go into it.

In this example 9 is the largest square factor.

$$\sqrt{54} = \sqrt{9 \times 6} \quad \text{now use the rule to split it.} \quad \sqrt{9 \times 6} \rightarrow \sqrt{9} \times \sqrt{6} \rightarrow 3\sqrt{6}$$

Adding or subtracting surds

You can only add 'like' surds – just as in algebra you can only add or subtract 'like' terms.

$$3\sqrt{2} + 5\sqrt{2} \rightarrow 8\sqrt{2} \quad \text{and} \quad 5\sqrt{3} - 2\sqrt{3} \rightarrow 3\sqrt{3} \quad \text{also:} \quad 3\sqrt{5} - \sqrt{5} \rightarrow 2\sqrt{5}$$

Sometimes we must simplify before we can add or subtract.

$$\sqrt{18} - \sqrt{2}$$

Simplify the $\sqrt{18}$ term by looking for the largest square factor.

$$\sqrt{18} - \sqrt{2} \rightarrow \sqrt{9 \times 2} - \sqrt{2} \rightarrow \sqrt{9} \times \sqrt{2} - \sqrt{2} \rightarrow 3\sqrt{2} - \sqrt{2} \rightarrow 2\sqrt{2}$$

$$\sqrt{63} - \sqrt{28}$$

Simplify both terms by looking for the largest square factor.

$$\sqrt{63} - \sqrt{28} \rightarrow \sqrt{9 \times 7} - \sqrt{4 \times 7} \rightarrow \sqrt{9} \sqrt{7} - \sqrt{4} \sqrt{7} \rightarrow 3\sqrt{7} - 2\sqrt{7} \rightarrow \sqrt{7}$$

Simplification

$$\sqrt{3} \times \sqrt{15}$$

Combine the two surds and look to simplify further.

$$\sqrt{3} \times \sqrt{15} \rightarrow \sqrt{3 \times 15} \rightarrow \sqrt{45} \rightarrow \sqrt{9 \times 5} \rightarrow \sqrt{9} \sqrt{5} \rightarrow 3\sqrt{5}$$

By combining the two surds – we can often find a square factor to simplify with.

Multiplying out brackets

$(2 + \sqrt{2})(3 + \sqrt{2})$ Just use FOIL and the rules of surds.

$$(2 + \sqrt{2})(3 + \sqrt{2}) \rightarrow 2 \times 3 + 2\sqrt{2} + 3\sqrt{2} + \sqrt{2}\sqrt{2} \rightarrow 6 + 5\sqrt{2} + 2 \rightarrow 8 + 5\sqrt{2}$$

More involved brackets:

$$\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) \rightarrow \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \sqrt{2} \frac{1}{\sqrt{2}} + \sqrt{2} \frac{1}{\sqrt{2}} - \sqrt{2}\sqrt{2} \rightarrow \frac{1}{2} - 1 + 1 - 2 \rightarrow 1\frac{1}{2}$$

Rationalising the denominator.

Surds are often in denominators and are sometimes inconvenient there.

We can remove them as follows:

e.g. $\frac{3}{\sqrt{2}}$ To rationalise this (i.e. get rid of the surd in the denominator)

Multiply top and bottom by the surd i.e. $\sqrt{2}$

$$\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{\sqrt{2} \times 2} \rightarrow \frac{3\sqrt{2}}{\sqrt{4}} \rightarrow \frac{3\sqrt{2}}{2}$$

This will work for any simple surd in the denominator.

However, we often have more involved surds such as $2 + \sqrt{2}$.

In this case we multiply by the conjugate (conjugate means opposite sign)

So the conjugate of $2 + \sqrt{2}$ is $2 - \sqrt{2}$

e.g. $\frac{1}{\sqrt{7}-1}$ multiply top and bottom by the conjugate $\frac{1}{\sqrt{7}-1} \rightarrow \frac{1}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1}$

$$\rightarrow \frac{\sqrt{7}+1}{(\sqrt{7}-1)(\sqrt{7}+1)} \rightarrow \frac{\sqrt{7}+1}{\sqrt{7}\sqrt{7} + \sqrt{7} - \sqrt{7} - 1} \rightarrow \frac{\sqrt{7}+1}{7 + \cancel{\sqrt{7}} - \cancel{\sqrt{7}} - 1} \rightarrow \frac{\sqrt{7}+1}{6}$$

Notice how the surd part always cancels out – difference of two squares, and the two surds multiply together to give a whole number.

You are using: $(a+b)(a-b) \rightarrow a^2 - b^2$